Math 120 Class Notes

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Definition 1 *Group.* A group $(G, \circ)$ is a pair of

1. a set G, and
2. a rule that takes in two elements $a, b \in G$ and outputs one element $a \circ b$. (a function $: G \times G \to G$)

satisfying the following axioms:

1. for any $x, y, z \in G$, $x \circ (y \circ z) = (x \circ y) \circ z$ (associative law)
2. there is an element $e \in G$ such that $x \circ e = e \circ x = x$ [Note: Unique. Can prove one order from the other.]
3. for every $x \in G$, there is an element $y \in G$ such that $x \circ y = y \circ x = e$ [Note: Unique.]

Note 1 *e is unique.* If e, e' both have the property of (2), $e' = e \circ e' = e$

Note 2 *Similarly, the y from* (3) *is unique.* It is called the *inverse* of x, denoted by x^{-1} (e.g. $e^{-1} = e$)

Example 1 *Some example groups:*

1. $G =$ all rotations of the sphere.
   $\circ$: $R \circ R' = R'$ followed by R e: rotation that does nothing
2. $G = \mathbb{R}$ (real numbers)
   $x \circ y = x + y$
   $e = 0$
   "x^{-1}"$= -x$
3. $G = \mathbb{R} \setminus \{0\}$
   $x \circ y = x \cdot y$
   $e = 1$
   $x^{-1} = 1/x$

4. Permutations. A *permutation* of X is a bijective function $X \to X$.
   If σ, σ' are permutations, then so is $\sigma \circ \sigma'$ = permutation given by first applying σ' then σ. (Think $(\sigma \circ \sigma')(x) = \sigma(\sigma'(x))$).
   For any set X, (set of permutations of X, $\circ$) is a group.

Example 2 *Take* $X = \{1, 2, 3\}$. There are $6 = 3!$ permutations of X.

Definition 2 *Symmetric group of* X. $Sym(X)$ = (permutations of X, $\circ$)
(Relationships between groups: section 1.6)

Definition 3 *Homomorphism.* $\varphi : (G\circ) \to (G', \circ')$ is a function $\varphi : G \to G'$ such that $\varphi(x \circ y) = \varphi(x) \circ' \varphi(y)$.