Math 109 Notes

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Non-abelian finite order
Symmetries of abe (composition)

Abelian infinite order
$(\mathbb{Z}, +)$ (countable)
$(\mathbb{Q}^{\times}, \cdot)$ (countable)
$(\mathbb{R}^{\times}, \cdot)$ (uncountable)

Suppose that G is a group with at least one non-identity element.
$G = \{e, g, g^{-1}, ...$
Possibilities for g:
1) $g = g^{-1}, gg = e$
2) $g \neq g^{-1}$

Def. Suppose $g \neq e$. The order of g is the smallest integer n such that $g^n = e$. $(\underbrace{ggg...g}_{n} = e)$

Suppose G is group with at least one non-identity element of infinite order.
$G = \{e, g, g^2, g^3...$
$\rightarrow$ Why is $g^n \neq g^m$ for $m \neq n$?
$(g^{-1})^n g^n = (g^{-1})^n g^m$
$\Rightarrow e = g^{-n+m}$
If $G = \{e, g, g^{-1} g^2, g^{-2}, g^3, g^{-3}...$ is everything, it is the
uninfinite cyclic group.

$(\mathbb{Z}, +)$ is an infinite cyclic group generated by 1 or -1.

Ex. Element with finite order: Symmetry group of a cube.
Consider a rotation r (y')
$rrrr = e$ implies order of r is 4.

Def. G and G' are <u>isomorphic</u> if there is a bijection $\phi : G \rightarrow G'$ such that $\phi(ab) = \phi(a)\phi(b) \wedge a, b \in G$

The infinite cyclic group generated by g is isomorphic to $(\mathbb{Z}, +)$

$(\mathbb{Z}_n, +)$
integers modulo n; a finite cyclic group of order n.
$\mathbb{Z}_n = \{0, 1, 2, ..., n-1\}$
$a + b = a + b$ mod 7

Dihedral groups D_n
For $n \geq 3$, this is the symmetry group of a regular n-gon.
$|D_n| = 2$
D_n is generated by two elements.
Triangle: Reflections a, b, c, rotate by $\frac{2\pi}{3} = r$ or $t = r^2$

	a	b	c	r	t	e
a	e					
b		e				
c			e			
r				t		
t						
e						

Presentation of the group D_3 using generators and relations. $D_3 = \{r, a | r^3 = 1, a^2 = 1, ar = r^{-1}a\}$

Show that any <u>word</u> in $< r, a, r^{-1}, a^{-1} >$ can be reduced to $\{e, r, r^2, a, ra, ra^2\}$ using the three relations.

$D_n = \{r, a | r^n = e, a^2 = e, ar = r^{-1}a\}$