Math 109 Notes

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A <u>set</u> is a collection of elements: - Integers in $\mathbb{Z}$
- students in Math 109.

Ex. $(\mathbb{Z}, +)$ form a group.
Adding two integers $\rightarrow$ integer
$+ : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Z}$
0 is special: $a \in \mathbb{Z} \equiv> a + 0 = 0 + a = a$
$\wedge a \in \mathbb{Z}, \exists$ an additive inverse $(-a)$

Ex. Symmetries of a Cube C
$\rightarrow$ Transformation sends C to itself.
r_1, r_2, r_3 (axes U, R, F), rotate each by $\frac{\pi}{2}, \pi, \frac{3\pi}{2}$
Reflect across the plane containing 2 r_i axes.
(Symmetries of C, composition)

Identity Symmetry: Do nothing
Inverses: rotation by $\phi \rightarrow$ rot $-\phi$, reflection $\rightarrow$ reflection

<u>Def</u>: A <u>group</u> is a set together with an associative operation $* : G \times G \rightarrow G$ such that - $\exists$ an identity $e \in G$ s.t. $g * e = e * g = g$ - $\wedge g \in G \exists$ an inverse g^{-1} with respect to $*$ s.t. $g * g^{-1} = g^{-1}g = 1$ - Associative: $a * (b * c) = (a * b) * c$

<u>Proposition</u>: Inverses are unique.
<u>Proof</u> Pick $g \in G$ and suppose f, h are inverses of g
$f = e * f$
$= (h * g) * f$ (because h is an inverse of g)
$= h * (g * f) = h * e = h$

<u>Prop</u>: The identity is unique
<u>Proof</u>: Suppose e, e' are identities in G: $e = e * e' = e'$
$g * e = g = g * e'$
$g^{-1} * (g * e) = g^{-1}(g * e)$
$(g^{-1} * g) * e = (g^{-1} * g) * e'$
$e * e = e * e'$

$e = e'$

Def The group $(G, *)$ is Abelian if $*$ is commutative $(a * b = b * a)$
Ex. $(\mathbb{Z}, +)$ are Abelian.

Let's find more groups!
$(Z, \cdot)$? - No inverse to, say, 3.
$(\mathbb{Q}, \cdot)$?
$(\mathbb{R}, \cdot)$?
$\mathbb{R}^x = \{x | x \neq 0 \in R\}$
$(\mathbb{R}, *)$ is an Abelian group.

How can we tell groups apart?
(Need invariants?)
Def The order of a cube is the number of elements in the set $|G|$.
$|\text{Symmetries of a Cube}| = 48$